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# Magnetic field dependence of the exciton bandwidth in diluted magnetic semiconductors with quantum wells

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**Abstract**

Effects of magnetic impurity concentration and spin-projection fluctuations on the exciton scattering, the exciton bandwidth, and the light reflection, transmission, and absorption spectra of diluted magnetic semiconductor crystals with quantum wells in an external magnetic field were studied. The fluctuations of the distributions of both single and pairs of impurity centres as well as the destruction of antiferromagnetically coupled pairs by a high magnetic field were taken into account. Calculations were performed for the quantum wells (Cd, Mn)Te/CdTe/(Cd, Mn)Te. It was shown, that: (i) the contribution of the exciton scattering to the exciton bandwidth is considerable—it is comparable with the experimentally observed bandwidths for the systems under study; (ii) the bandwidth increases as the magnetic field rises for the $\sigma^-$-component of the exciton transition, while the bandwidth decreases for the $\sigma^+$-component. In particular, the exciton scattering probability drops abruptly when the total magnetic moment of the antiferromagnetically bound pair appears, due to the magnetic field-induced destruction of the pairs. The destruction of the pairs leads to narrowing of the $\sigma^+$-component of the exciton transition. These peculiarities in the magnetic field dependence of the exciton bandwidth were explained by the coherent summation of spin-dependent and spin-independent parts of the interaction between the exciton and the impurities.

## 1. Introduction

Optical band parameters (broadening, intensity, etc) of crystals with quantum wells determine the role which these structures play in different types of device. The band broadening is determined essentially by the technological conditions of the crystal growth, namely the presence of defects, interface structure, inhomogeneities, strain effects, etc. However, the overwhelming majority of low-dimensional quantum systems are grown from a basis of semiconductor alloys and the compositional fluctuations of these alloys must contribute considerably to the optical bandwidth. The effect of the compositional fluctuations on the

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excitonic spectra has been studied in [1, 2], where it was shown that the contribution of the fluctuations to the excitonic band broadening is important.

In diluted magnetic semiconductors the strong exchange interaction between the charge carriers and the localized magnetic ions leads to significant changes in the energy positions of the exciton lines in the external magnetic field. Therefore the fluctuations of the magnetic impurity distribution must have a strong influence on the magnetic field dependence of the optical band parameters. In most of the analyses of the excitonic spectra of diluted magnetic semiconductor crystals, the impurity distribution has been assumed to be homogeneous. Compositional fluctuations lead to inhomogeneous impurity distributions like that shown, as an example, in figure 1. Such an inhomogeneous distribution must result in quantitative changes of the band position as well as in the band broadening due to the exciton scattering on the compositional fluctuations. The diluted magnetic semiconductors are particularly attractive for study because in these crystals the bandwidth change must depend on the external magnetic field, which offers the possibility of tuning the exciton band parameters through the application of magnetic fields. In contrast to the case for non-magnetic semiconductors, in the semimagnetic alloys the band broadening is caused not only by the impurity concentration fluctuations, but also by the impurity spin-projection fluctuations. In the present paper the effect of the impurity concentration and spin-projection fluctuations on the optical bandwidths in the reflection and absorption spectra of diluted magnetic semiconductors containing quantum wells is analysed.

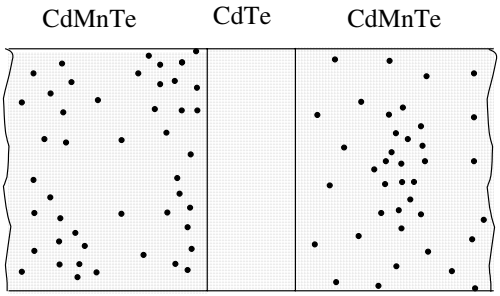


**Figure 1.** A schematic illustration of inhomogeneous magnetic ion distribution in the heterostructure CdMnTe/CdTe/CdMnTe.

To study the band broadening and the band shift due to the fluctuations, we have used a method developed for bulk materials: in [3] for the Frenkel excitons in molecular crystals with isotopic impurities and in [4] for the Wannier excitons in semiconductors with charged impurities. According to this method, the probability of exciton scattering on the impurity concentration fluctuations is studied and the exciton lifetime with respect to the scattering determines the width of the exciton band. In the zero-order approximation the interaction of the exciton with the impurities is calculated with the assumption of a uniform impurity distribution with the average concentration. Deviations of the concentration from the average value (fluctuations) lead to exciton scattering, which influences the exciton bandwidth.

We will show that the contribution of the fluctuations to the bandwidth is considerable; it depends strongly on the magnetic field—moreover, the width of some bands decreases as the magnetic field rises. Unlike in the previous paper [5], in the present one we have taken into account the exciton scattering on the pairs of antiferromagnetically coupled spins, have studied absorption spectra, and have used realistic values of parameters for (Cd, Mn)Te/CdTe/(Cd, Mn)Te quantum wells. We would like to note that we have not taken into account the exciton band narrowing due to magnetic polaron formation, because magnetic polaron effects do not occur in reflection and absorption [6].

## 2. Exciton scattering on the magnetic impurity and spin-projection fluctuations in diluted magnetic semiconductor quantum wells—magnetic field-induced suppression of the fluctuations

Let us consider a (Cd, Mn)Te/CdTe/(Cd, Mn)Te quantum well. In this case the magnetic impurities are localized in the barrier layers. The Hamiltonian of the exciton in the system under study can be written as

$$H = H_0 + H_{int} \tag{1}$$

where $H_0$ is the traditional free-exciton Hamiltonian, including the kinetic energy of the electron and the hole, the Coulomb interaction, and the interaction of the exciton with the external magnetic field; $H_{int}$ describes the interaction of the carriers with the magnetic impurities. Here $H_{int}$ consists of spin-dependent and spin-independent parts. The Hamiltonian of the interaction of the exciton with the magnetic ions is given by

$$H_{int} = \sum_{\vec{n}} \frac{1}{N_0} \left[ (\Delta_e - J_e \vec{S}_e \cdot \vec{S}_{\vec{n}}) \delta(\vec{r}_e - \vec{n}) + (\Delta_h + J_h \vec{S}_h \cdot \vec{S}_{\vec{n}}) \delta(r_h - \vec{n}) \right] x_{\vec{n}} \tag{2}$$

where $N_0$ is the concentration of cationic lattice sites, $\vec{n}$ is the coordinate of the cationic lattice site, $\vec{r}_{e\,(h)}$ is the position of the electron (hole), $\Delta_{e\,(h)}$ is the potential of the non-magnetic interaction of the electron (hole) with the impurity ion, $J_{e\,(h)}$ is the exchange integral for the electron (hole), $\vec{S}_{e\,(h)}$ is the spin of the electron (hole), $\vec{S}_{\vec{n}}$ is the spin of the magnetic ion, $x_{\vec{n}} = 0$ if there is a $Cd^{2+}$ ion at the lattice site $\vec{n}$, and $x_{\vec{n}} = 1$ if the site is occupied by a $Mn^{2+}$ ion.

Since the exciton radius in the semiconductors studied exceeds considerably the distance between the impurity centres for typical concentrations, the exciton interacts with a large number of the impurities. Therefore, using the traditional mean-field approximation the Hamiltonian (2) is replaced by the averaged one, where $x_{\vec{n}}$ and $\vec{S}_{\vec{n}}$ are replaced by their average values. But since expression (2) contains the product $x_{\vec{n}} \vec{S}_{\vec{n}}$, the values of $x_{\vec{n}}$ and $\vec{S}_{\vec{n}}$ are independent only for the single-impurity centres even for statistically equiprobable distribution of the impurities. The point is that there are clusters of antiferromagnetically paired spins besides single isolated ions $Mn^{2+}$ in the diluted magnetic semiconductor [7]. Due to the antiferromagnetic interaction, the spins of the nearest-neighbouring magnetic ions are oriented in antiparallel directions in the weak magnetic field and at low temperatures. In the mean-field approximation these coupled spins do not make a contribution to the magnetization of the crystal, but their influence becomes apparent in strong fields. Thus, there is a strong correlation of the values of $x_{\vec{n}}$ and $\vec{S}_{\vec{n}}$ for the magnetic ions at the nearest-neighbour sites and therefore it is worthwhile to split the Hamiltonian (2) into parts describing the interaction of the exciton with the singles, pairs, triples and higher-order centres. After the regrouping the Hamiltonian (2) can be written as

$$H_{int} = \sum_{\vec{n}} \frac{1}{N_0} \Big\{ [(\Delta_e - J_e \vec{S}_e \cdot \vec{S}^{(1)}_{\vec{n}}) \delta(\vec{r}_e - \vec{n}) + (\Delta_h + J_h \vec{S}_h \cdot \vec{S}^{(1)}_{\vec{n}}) \delta(r_h - \vec{n})] x^{(1)}_{\vec{n}} + \frac{1}{2} [(2\Delta_e - J_e \vec{S}_e \cdot \vec{S}^{(2)}_{\vec{n}}) \delta(\vec{r}_e - \vec{n}) + (2\Delta_h + J_h \vec{S}_h \cdot \vec{S}^{(2)}_{\vec{n}}) \delta(r_h - \vec{n})] x^{(2)}_{\vec{n}} \Big\} \tag{3}$$

where $\vec{S}^{(1)}_{\vec{n}}$ is the spin of the single ion, $\vec{S}^{(2)}_{\vec{n}}$ is the total spin of the pair, $x^{(1)}_{\vec{n}}$ describes the occupation of the cation sites by the single impurity ions, $x^{(2)}_{\vec{n}}$ describes the belonging of the site to the cluster. Here we assume that for the low impurity concentration, the probability of the formation of triples and higher-order clusters is small. We also neglect the interaction of the single ions with the paired ones. In the Hamiltonian (3), $\vec{S}^{(1)}_{\vec{n}}$ fluctuates independently of $x^{(1)}_{\vec{n}}$, and $\vec{S}^{(2)}_{\vec{n}}$ fluctuates independently of $x^{(2)}_{\vec{n}}$. The coefficient $1/2$ in the second item of

(3) appears because the cluster is counted twice due to the summation over $\vec{n}$. In (3) we also assume that the exciton wave function changes slightly within the limits of the cluster.

The numbers of sites occupied by single and paired ions can be expressed as

$$x_{\vec{n}}^{(i)} = x^{(i)} + \delta x_{\vec{n}}^{(i)} \qquad S_{z,\vec{n}}^{(i)} = \bar{S}_z^{(i)} + \delta S_{z,\vec{n}}^{(i)}. \tag{4}$$

Here $i = 1, 2$, $x^{(1)}$ and $x^{(2)}$ denote the average values of the relative concentrations of the single and paired ions, $\bar{S}_z^{(1)}$ is the average value of the single-spin projection on the direction of the magnetic field (along the $z$-axis), $\bar{S}_z^{(2)}$ is the average value of the spin projection of the pair, and $\delta x_{\vec{n}}^{(1)}$, $\delta x_{\vec{n}}^{(2)}$, $\delta S_{z,\vec{n}}^{(1)}$, and $\delta S_{z,\vec{n}}^{(2)}$ denote the fluctuations of the corresponding values. For the isolated spin,

$$\bar{S}_z^{(1)} = -S B_{5/2}\left[\frac{\frac{5}{2} g \mu_B H}{k_B T}\right]$$

where

$$B_{5/2}(y) = \frac{6}{5}\coth\left(\frac{6}{5}y\right) - \frac{1}{5}\coth\left(\frac{y}{5}\right)$$

is the Brillouin function, $S = \frac{5}{2}$, $k_B$ is the Boltzmann constant, $T$ is the temperature. The expression for $\bar{S}_z^{(2)}$ is given below.

Neglecting the clusters containing more than two antiferromagnetically coupled ions, we assume that in the crystal with the average relative impurity concentration $x$ the average relative concentration of the single ions is equal to $x^{(1)} = x(1-x)^N \approx x(1-Nx)$ and that for paired ions is equal to $x^{(2)} = Nx^2$; $N = 12$ is the number of nearest-neighbouring cationic sites [7].

Using (4), the Hamiltonian (1) can be written as

$$H = \bar{H}_0 + \Delta H$$

where $\bar{H}_0 = H_0 + \bar{H}_{int}$; $\bar{H}_{int}$ is the Hamiltonian of the interaction between the exciton and the impurities in the quantum well with the semimagnetic barriers, in the mean-field approximation, and $\Delta H$ describes the fluctuations:

$$\bar{H}_{int} = \sum_{i=1,2} (x^{(i)} V_e^{(i)} \Theta(|z_e| - L/2) + x^{(i)} V_h^{(i)} \Theta(|z_h| - L/2)) \tag{5}$$

$$\Delta H = \sum_{\vec{n}} \Bigg\{ x^{(1)} V\, \delta S_{z,\vec{n}}^{(1)} + W^{(1)}\, \delta x_{\vec{n}}^{(1)} + V\, \delta S_{z,\vec{n}}^{(1)}\, \delta x_{\vec{n}}^{(1)} + \frac{1}{2}\left[ x^{(2)} V\, \delta S_{z,\vec{n}}^{(2)} + W^{(2)}\, \delta x_{\vec{n}}^{(2)} + V\, \delta S_{z,\vec{n}}^{(2)}\, \delta x_{\vec{n}}^{(2)} \right] \Bigg\} \tag{6}$$

where

$$V = -\frac{1}{N_0}(J_e S_{z,e}\delta(\vec{r}_e - \vec{n}) - J_h S_{z,h}\delta(\vec{r}_h - \vec{n}))$$

$$W^{(i)} = \frac{1}{N_0}(V_e^{(i)}\delta(\vec{r}_e - \vec{n}) + V_h^{(i)}\delta(\vec{r}_h - \vec{n}))$$

$$V_{e\,(h)}^{(1)} = (\Delta_{e\,(h)} \mp J_{e\,(h)} S_{z,e\,(h)} \bar{S}_z^{(1)})$$

$$V_{e\,(h)}^{(2)} = \frac{1}{2}(2\Delta_{e\,(h)} \mp J_{e\,(h)} S_{z,e\,(h)} \bar{S}_z^{(2)}).$$

As has been mentioned above, when the impurity concentration is high, the exciton interacts with a large number of ions and 'feels' their average concentration. Therefore for the analysis

of the exciton spectra the exciton–impurity interaction in the form $\bar{H}_{int}$ is usually used. In the present paper we have taken into account fluctuations, which are described by $\Delta H$.

For calculations we have used the following form of the variational wave function of the confined exciton in the quantum well with the Hamiltonian $\bar{H}_0$:

$$\Psi_{\vec{k}}(\vec{r}_e, \vec{r}_h) = \frac{1}{\sqrt{S}} e^{i\vec{k}\cdot\vec{R}} \Phi(\vec{\rho}, z_e, z_h) \tag{7}$$

$$\Phi(\vec{\rho}, z_e, z_h) = f_e(z_e) f_h(z_h) \sqrt{\frac{2}{\pi\lambda^2}} e^{-\rho/\lambda} \tag{8}$$

where $f_{e\,(h)}(z_{e\,(h)})$ is the wave function of the lowest electron (hole) subband in the quantum well, $\vec{r}_{e\,(h)} = (\vec{\rho}_{e\,(h)}, z_{e\,(h)})$, $\vec{k}$ and $\vec{R}$ are the wave vector and the position of the exciton centre of mass in the plane of the layers, $z$ is the direction of crystal growth and of the applied magnetic field. The fluctuations lead to exciton scattering with change of the two-dimensional wave vector $\vec{k}$. Since the exciton bandwidth greatly exceeds the energy of the exciton–impurity interaction, the perturbation theory approximation may be used to calculate the relaxation time $\tau_{\vec{k}}$:

$$\frac{\hbar}{\tau_{\vec{k}}} = \sum_{\vec{k}'} W_{\vec{k},\vec{k}'} \tag{9}$$

where $W_{\vec{k},\vec{k}'}$ is the probability of the exciton scattering from the $\vec{k}$-state to the $\vec{k}'$-state calculated with the wave functions (7), (8) and perturbation Hamiltonian (6). We would like to note that the spin-dependent scattering of the conduction electrons on magnetic ions in bulk semimagnetic crystals was investigated in [8].

The relaxation time obtained was averaged with respect to impurity concentration and spin-projection distributions assuming these distributions to be chaotic:

$$\begin{aligned} &\langle \delta x^{(1)}_{\vec{n}}, \delta x^{(1)}_{\vec{m}} \rangle = x^{(1)}(1 - x^{(1)})\delta_{\vec{n},\vec{m}} \\ &\langle \delta x^{2}_{\vec{n}}, \delta x^{(2)}_{\vec{m}} \rangle = x^{(2)}\delta_{\vec{n},\vec{m}} \\ &\langle \delta S^{(i)}_{z,\vec{n}}, \delta S^{(i)}_{z,\vec{m}} \rangle = \langle (\delta S^{(i)}_{z})^2 \rangle \delta_{\vec{n},\vec{m}}. \end{aligned} \tag{10}$$

Below, we will consider the cases of weak and high magnetic fields separately. Here 'weak field' means a field which is insufficient to unlock the antiferromagnetically bound pairs.

### *2.1. Weak magnetic fields*

It is well known that in diluted magnetic crystals in a weak magnetic field at low temperature the spin of the pairs of nearest-neighbour magnetic ions is compensated due to the antiferromagnetic interaction. Because of this, the average value of the total spin of such a pair, $\bar{S}^{(2)}_z$, in (4), (5) is equal to zero. Therefore the spin-independent part of the interaction of the carriers with all of the magnetic ions contributes to the linewidth studied, while the spins of the single ions only result in exciton scattering.

Calculations of the value $\hbar/\tau_E$, where $E$ is the kinetic energy of the exciton movement in the plane of the heterostructure layers, were performed numerically for the CdMnTe/CdTe/CdMnTe heterostructure with $x = 0.05$ and $T = 2$ K. The following parameters were used [9]: the total-energy band-gap discontinuity $\mathrm{d}E_g(x) = 1.592x$ eV, $\Delta_e = (1 - \tilde{\alpha})\ \mathrm{d}E_g/\mathrm{d}x$, $\Delta_h = \tilde{\alpha}\ \mathrm{d}E_g/\mathrm{d}x$, $\tilde{\alpha} = 0.4$; the effective masses of the electron and the heavy hole are equal to $m_e = 0.096\ m_0$, $m_{hh} = 0.64\ m_0$, where $m_0$ is the mass of the free electron; the dielectric constant $\varepsilon = 9.7$, $J_e = 0.22$ eV, $J_h = -0.88/3$ eV.

The calculated magnetic field dependence of the reverse relaxation time $\hbar/\tau_E$ for $E = 0$ is depicted in figure 2. The broadening of the excitonic band caused by the scattering mechanism studied depends essentially on the magnetic field intensity. Also, the magnetic dependence is different for the $\sigma^-$- and $\sigma^+$-components of the exciton transition: $\hbar/\tau_E$ increases as the field rises in the case of the $\sigma^-$-transition and decreases as the field increases for the $\sigma^+$-transitions. To explain this result it should be noted that for the spin orientation corresponding to the $\sigma^-$-component the spin-dependent part of the exciton–impurity interaction adds to the spin-independent one, while for the $\sigma^+$-component they tend to compensate each other. In other words, there is a magnetic field-induced suppression of the fluctuation potential and a corresponding narrowing of the $\sigma^+$-component of the exciton lines in the diluted magnetic semiconductor quantum wells.

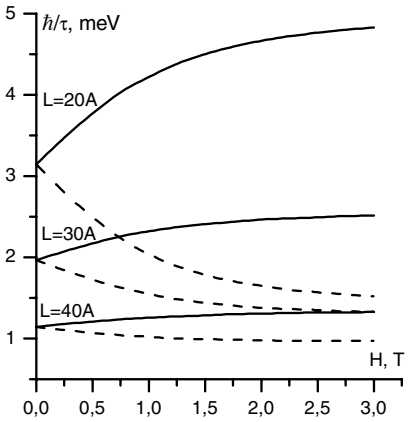


**Figure 2.** The reverse relaxation time for the $\sigma^-$-polarization (solid line) and $\sigma^+$-polarization (dashed line) as functions of the magnetic field at $E = 0$ for various values of the well width and $x = 0.05$.

This effect increases with the decrease of the well width because the probability of wavefunction penetration into barrier layers containing the scattering magnetic impurity rises. It should be noted that some experimental evidence of excitonic band narrowing with increase of the magnetic field intensity in bulk semimagnetic semiconductors was reported in [10]. As was mentioned above, in the semimagnetic crystal the magnetic polaron formation leads to the exciton luminescence bandwidth narrowing [6], but this effect does not occur in the reflection and transition spectra, which are studied in our paper. In the case studied, the line narrowing is connected with a different mechanism: competition between the spin-dependent and spin-independent interaction of the exciton with the magnetic impurities. It is possible that the mechanism that we considered takes place also in luminescence spectra. But comparison of the contribution of this mechanism to the linewidth with the contribution of the polaron effect needs additional investigation.

### *2.2. High magnetic fields*

Interesting features are observed in the magnetic field dependence of the relaxation time in high magnetic fields. As noted above, in low fields the spin-dependent interaction contributes to the linewidth only for single magnetic ions, because the average spin of the pairs is vanishing. For the $\sigma^+$-transition, the potential of the exchange interaction of the carriers with the single impurities compensates the non-magnetic interaction more and more as the magnetic field $H$ rises, which results in the decreasing of the exciton scattering probability $\hbar/\tau_{\vec{k}}$. At $H > 3$ T the magnetization of the single ions becomes saturated and $\hbar/\tau_{\vec{k}}$ is practically unchanged as $H$ rises (figure 3). Further increase of the magnetic field intensity leads to the destruction of

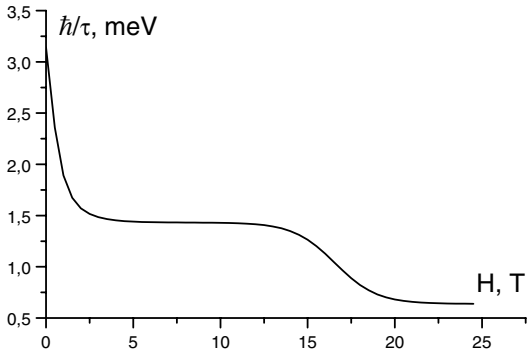


**Figure 3.** The magnetic field dependence of the reverse relaxation time for the $\sigma^+$-component of the exciton transition for $E = 0$ in the quantum well with $L = 20$ Å, $x = 0.05$, $T = 1.6$ K.

the antiferromagnetically coupled pairs of spins and step-like change of the average value of the pair spin projection as well as the sample magnetization. Thus in high magnetic fields pair centres begin to contribute to the exciton scattering.

In order to calculate the average spin projection of a pair, we have taken into account that the Hamiltonian of the pair of magnetic ions coupled by the Heisenberg interaction in the magnetic field $H$ has the following energy levels [7]:

$$E(S_T, m, H) = -\frac{1}{2}J_1[S_T(S_T+1) - 2S(S+1)] + g\mu_B m H.$$

Here, $S = \frac{5}{2}$, $m = -S_T, -S_T+1, \ldots, S_T$, $0 \leqslant S_T \leqslant 2S$, $J_1 = -22.2$ K [11] is the nearest-neighbour exchange constant, $g = 2$ is the $g$-factor of the $Mn^{2+}$ ions, $\mu_B$ is the Bohr magneton. Thus, the average spin projection for the pair is given by

$$\bar{S}_z^{(2)} = \left[\sum_{S_T=0}^{5}\sum_{m=-S_T}^{S_T} m\exp\left(-\frac{E(S_T,m,H)}{k_BT}\right)\right] \Big/ \left[\sum_{S_T=0}^{5}\sum_{m=-S_T}^{S_T}\exp\left(-\frac{E(S_T,m,H)}{k_BT}\right)\right].$$

In figure 3 we display the results of the relaxation time calculations for the exciton scattering on the both single and pair centres. The first magnetization step connected with the antiferromagnetic pair destruction occurs at $H^{(1)} = |J_1|/g\mu_B = 16.5$ T. In all, the pair magnetization has five steps with $H^{(n)} = nH^{(1)}$, $n = 1, 2, 3, 4, 5$ [7]. This leads to the step-like decrease of the exciton scattering probability and, consequently, the bandwidth of the $\sigma^+$-component of the exciton transition.

Thus, we again see an interesting phenomenon, where the inclusion of the new scattering mechanism results in a decrease of the scattering probability. As has been explained above for the scattering on the single centre, this is connected with the coherent summation of two interactions: spin dependent and spin independent.

The narrowing of the exciton luminescence bands and, particularly, the step-like decrease of the bandwidth in high magnetic field were observed experimentally in the semimagnetic quantum wells ZnSe/Zn(Cd, Mn)Se [11].

## 3. The effect of compositional fluctuations on reflection and absorption spectra in diluted magnetic semiconductor quantum wells; magnetic field-induced narrowing of the exciton band $\sigma^+$-component

Let us consider the effect of the magnetic impurity concentration and spin-projection fluctuations on the spectra of the light reflection and the absorption by the quantum well with semimagnetic barriers. The calculations are simplified if the well width is smaller than

the wavelength, which holds true for most cases. This approximation was used in [12], where the electromagnetic wave damping in a crystal was taken into account phenomenologically. Below, we calculate the reflection and absorption coefficients of the light in the quantum well, taking into account the frequency dependence of the damping due to the exciton scattering on the fluctuations.

We will suggest that the light wave frequency is close to the resonance frequency of the exciton transition in the quantum well. Using traditional theory of the interaction of the electromagnetic field with a system, we will find the current of polarization connected with the creation of the excitons for normal incidence for a linear approximation to the intensity of the electric field:

$$\vec{J}(z) = \mathrm{i}\frac{e^2|\vec{p}_{cv}|^2}{m_0^2\omega\hbar}\left(\Phi(0,z,z)\int \Phi(0,z',z')\vec{E}(z')\,\mathrm{d}z'\right)\Big/[\omega-\omega_0+\Delta(\omega)+\mathrm{i}\Gamma(\omega)] \tag{11}$$

where $\omega$ and $\omega_0$ are the light and resonance frequencies respectively, $\vec{p}_{cv}$ is the interband matrix element of the momentum, $\Gamma(\omega)$ is the exciton damping, $\Delta(\omega)$ is the resonance frequency shift, caused by the exciton scattering, $\Gamma(\omega) = \Gamma_0 + 1/[2\tau(\omega)]$, $\tau(\omega)$ is the relaxation time (9) for the exciton due to the scattering under study, $\Gamma_0$ takes into account other mechanisms of wave damping.

The current density (11) is non-zero in the vicinity of the quantum well. If the well width is considerably smaller than the wavelength, the influence of the quantum well may be taken into account via introduction of the surface polarization current [13]:

$$\vec{I} = \sigma_{surf}\vec{E} \tag{12}$$

where

$$\vec{I} = \int \vec{J}(z)\,\mathrm{d}z$$

$$\sigma_{surf} = \mathrm{i}\frac{\omega_{LT}\omega\varepsilon a_B^3}{4}\left(\left|\int \Phi(0,z,z)\,\mathrm{d}z\right|^2\right)\Big/[\omega-\omega_0+\Delta(\omega)+\mathrm{i}\Gamma(\omega)].$$

Here, $\sigma_{surf}$ is the surface conductivity, $\omega_{LT}$ is the longitudinal–transverse splitting, $\varepsilon$ is the dielectric constant, $a_B$ is the bulk Bohr radius.

To determine the coefficients of the amplitude reflection $r_{QW}$, the transmission amplitude $t_{QW}$, and the absorption $A$, one can use the boundary conditions for the Maxwell equations in the presence of surface conductivity (12) [13]:

$$\begin{aligned} r_{QW} &= \frac{(4\pi/c)\sigma_{surf}}{2n+(4\pi/c)\sigma_{surf}} = \frac{\mathrm{i}\Gamma_0}{\omega-\omega_0+\Delta(\omega)+\mathrm{i}\Gamma_0+\mathrm{i}\Gamma(\omega)} \\ t_{QW} &= \frac{2n}{2n+(4\pi/c)\sigma_{surf}} = \frac{\omega-\omega_0+\Delta(\omega)+\mathrm{i}\Gamma(\omega)}{\omega-\omega_0+\Delta(\omega)+\mathrm{i}\Gamma_0+\mathrm{i}\Gamma(\omega)} \end{aligned} \tag{13}$$

where

$$\begin{aligned} \Gamma_0 &= \left(\pi\omega_{LT}\omega\sqrt{\varepsilon}a_B^3\left|\int \Phi(0,z,z)\,\mathrm{d}z\right|^2\right)\Big/(2c) \\ A &= 1-|r_{QW}|^2-|t_{QW}|^2. \end{aligned} \tag{14}$$

Here $n$ is the bulk refraction coefficient.

The amplitude reflection coefficient for reflection from CdMnTe crystal containing a quantum well at a distance $d$ from the crystal surface can be written as

$$r = \frac{r_0 + r_{QW}\mathrm{e}^{\mathrm{i}\varphi}}{1 + r_0 r_{QW}\mathrm{e}^{\mathrm{i}\varphi}} \tag{15}$$

where

$$\varphi = \frac{\omega}{c}\sqrt{\varepsilon}(2d + L).$$

$r_0$ is the reflection coefficient at the surface of the crystal and $L$ is the well width.

The following parameters for the calculations of the reflection $R = |r|^2$ and absorption spectra for normal incidence of light for the crystal containing a quantum well were used: $\hbar\omega_{LT} = 1.25$ meV [12], $\hbar\Gamma_0 = 0.25$ meV, the energy gap for CdTe $E_g = 1.606$ eV [9].

The dependence of the normal-incidence reflection coefficient on the incident light frequency for the CdMnTe/CdTe/CdMnTe quantum well with the width $L = 30$ Å and $x = 0.05$ is shown in figure 4. It can be seen from this figure that the scattering studied leads to an appreciable change of the bandwidth and to a resonance energy shift, which depend on the magnetic field.

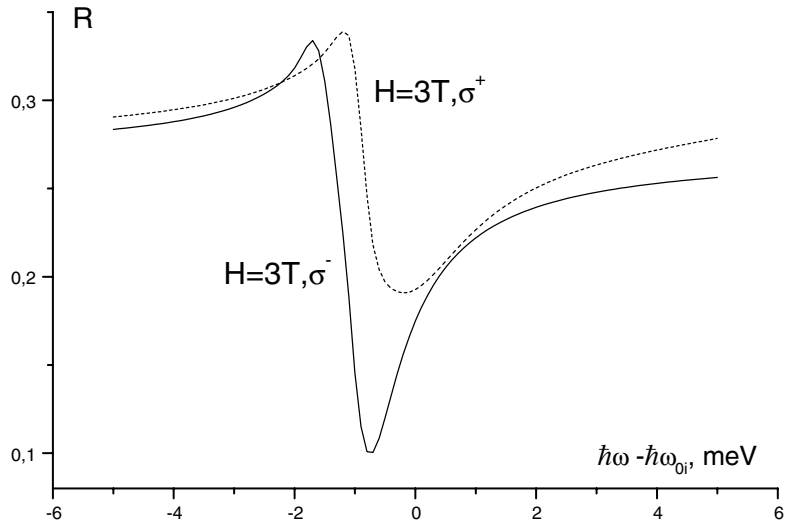


**Figure 4.** Normal-incidence reflection spectra for the quantum well with $L = 30$ Å, $x = 0.05$, $d = 413$ Å ($\omega_0^i$ is the exciton resonance frequency in the magnetic field $H = 3$ T: $\omega_0^-$ for the $\sigma^-$-transition and $\omega_0^+$ for the $\sigma^+$-transition).

The effect of the magnetic impurity concentration and spin-projection fluctuations manifests itself more clearly in the absorption spectra of the quantum well layer. In figure 5 the broadening of the $\sigma^-$-component and the narrowing of the $\sigma^+$-component of the excitonic band can be seen; the effect depends considerably on the well width.

## 4. Conclusions

In this paper we have considered the simplest mechanism of exciton scattering, which is always operative in alloys: the exciton scattering on the fluctuations of the impurity concentration and spin projection. There are inhomogeneities of the impurity distribution and defects of technological character, which result in band broadening too. However, the calculations performed in this paper show that the contribution of these fluctuations to the broadening is comparable with the experimentally observed bandwidths. Thus the compositional fluctuations are an important cause of exciton band broadening.

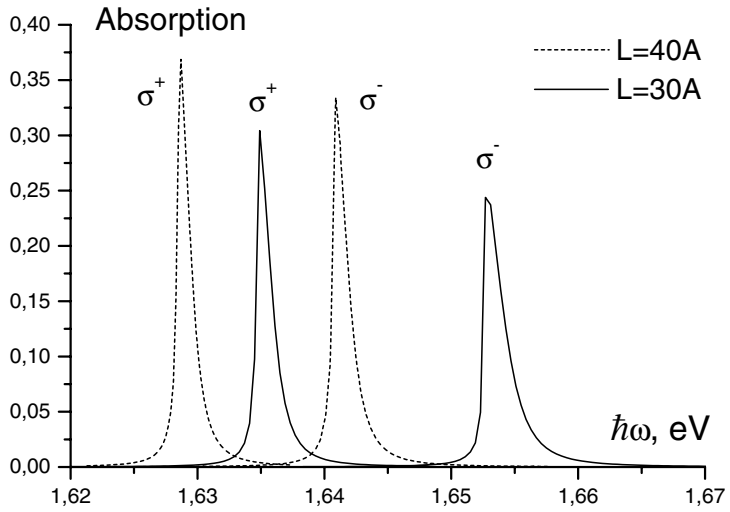


**Figure 5.** Normal-incidence absorption spectra for the quantum wells with $L = 30$ Å and $L = 40$ Å; $x = 0.05$, $H = 3$ T.

We should note the distinguishing feature which appears in the magnetic dependence of the band broadening in the diluted magnetic semiconductors. There is coherence in the exciton scattering on the spin-dependent and spin-independent parts of the interaction of the exciton with the magnetic impurities. As the result, the bandwidth increases as the magnetic field increases for the $\sigma^-$-component of the spectra, while it decreases for the $\sigma^+$-component. In particular, if an antiferromagnetically coupled pair is destroyed by the magnetic field, this gives rise to a magnetic interaction of the exciton with the pair which compensates the spin-independent interaction and, therefore, leads to a decrease of the width of the $\sigma^+$-component. Therefore, we may conclude that it is possible to tune the bandwidth through the application of a magnetic field to diluted magnetic semiconductors.

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